All math concepts in this note relates to, but is not actually what they are.

We start with an item, and two functions, step up and its reverse, step down, which I'll call $u(x)$ and $d(x)$.
In reality $u(x)$ is just $x+1$ but to keep the essence I will describe it as an operation that returns an another item that is unique to the input.
And we start with an arbitrary item (0), using recursion we can get $\mathbb{Z}$

In this case, 0 is the constant of step functions $u(num) = n\sim$
$d(num) = \sim n$

Then is addition, repetition of step up, and subtraction, repetition of step down. Notice subtraction is the same as addition, but with step down instead.
e.g. $3+2=5$, $3-2=1$; $num\underbrace{+a}_{\text{addition}}$, $num\underbrace{-a}_{\text{subtraction}}$

Now multiplication, instead of the normal definition, it's more like this:
$num\underbrace{+a\cdot b}_{\text{multiplication}}$ and $num\underbrace{-a\cdot b}_{\text{"division"}}$

where b is the number of times to add

OOC:
Wait I see something now, it is like surds and sometimes we don't get surds in expressions by luck and sometimes we don't get fractions by luck.

exponent:
$num+a\cdot b\cdot c$ $num-a\cdot b\cdot c$
and c will be the "exponent"

So if defined like this, the hyperoperation series would just become addition of a product with increasing dimensions, maybe because I did not reference num in the recursion?

so now, recursing only (repeating previous operation num times)

[num] = immutable copy of num at the beginning of operation

			$n=$
u(num) num times (	u (num)	d (num)) d (num) num times	0
	2·num	0·num	1
	num+[num]	num−[num] wait	
	num·(1+1)	num(1−1)	

but the numbers shouldn't actually represent numbers in my original idea, but simply discrete, unique "tags".
Anyways continuing with idea of actual numbers.

$\underbrace{\ldots\ \overbrace{u(u(\ldots u(num)\ldots))}^{[num]\text{ times}}}_{[num]\text{ times}}$

		n
num+[num]·[num]	num−[num]·[num]	2
…	…	…
num+[num]^n	num−[num]^n	n

still same as before...

In reality:

$num + a \Rightarrow \underbrace{u(num)}_{a \text{ times}}$ $num + [num]$

$num \times a \Rightarrow \overbrace{num + num}^{a \text{ times}}$ $num + [num] \cdot ([num] - 1)$ — due to different definition ?

So replace the final [num] with a? works for multiply

$num^a \Rightarrow \overbrace{num \cdot num}^{a \text{ times}}$ $num + [num]([num] - 1) \cdot a$

$2^3: 2 + 2 \cdot 3 = 2 + 6 = 8$ $2^2: 2 + 2 \cdot 2 = 6$ ×

$3^2: 3 + 3 \cdot 2 \cdot 2 = 3 + 12 = 15$ ×

$num + [num]$

repeat twice

$(num + [num]) + (num + [num])$

so this is the actual recursion?

square brackets doesn't mean anything now

so addition is num + a

multiplication: $b \cdot (num + a) = b \cdot num + ab$

exponential: e.g.

$b \cdot (b\,num + a + a)$ $c = 2$

OR $b \cdot b\,num + ab^2 + ab$ 2

$\therefore$ exponential is: $b^c \cdot num + \sum_{n=1}^{c} a b^n$ $ab \cdot b^{n-1}$

$= b^c \cdot num + \frac{ab \cdot (b^c - 1)}{b - 1}$

random:

addition is different than upper operations as it starts from the number being added instead of 0 or 1

i.e. to get the above derived to their actual counter part, for addition num = num, but for multiplication num = 0 and exponential num = 1, a = 0

~~hypothesis~~ conjecture: If this continues on, for tetration do we use num = 2, a = 1, b = 0 to get actual?

So actually, hyperoperation sequence is:

$n=-1$ constant
$n=0$ step (uniary function)
$n=1$ binary function
$n=2$ trinary function

Oh yeah the preexisting definitions are "flawed",
let me redefine my own:

My own hyperoperation sequence (H_n) is a sequece of functions, increasing in number of inputs, that are recusion of the previous function by the new input by the input "num" (future question: what if instead of num, let's branch out at each term for recursing "a", "b", etc?) Spoiler alert: yes

conventional

$n=0$ constant ($H_0(num) = num$)

$n=1$ stepper ($H_1(num) = num+1$)

$n=1$ is not a recursion of $n=0$

$H_1(num) = u(num)$

$n=2$ $H_2(num, a) = num+a$

$n=3$ $H_3(num, a, b) = b \cdot num + a \cdot b$

$n=4$ $H_4(num, a, b, c) = b^c \cdot num + \frac{a \cdot b(b^c - 1)}{b-1}$

So actually, instead of functions, think of it as a relationship relating n+1 variables (output counts as well)
And there are functions to find one variable with all other input
And perhaps rename num to just a, but output to Ω
So better definition is

H_0, constant, just the variable itself $H_0\{a\}$ ← just made this up, represent constraint
H_{1a}, stepper, $H_2\{a,\Omega\}$ is satisfied if $u(a)=\Omega$
H_{2a} $H_3\{a,b,\Omega\}$ if $a+b=\Omega$
H_{3a} $H_4\{a,b,c,\Omega\}$ if $ac+bc=\Omega$
H_{4a} $H_5\{a,b,c,d,\Omega\}$ if $a\cdot c^d + bc\cdot\frac{(c^d-1)}{c-1}=\Omega$

starting from H_{2a}, all further terms are recursing the output (Ω) back into a a new variable times.

instead of a b c d, ... etc. perhaps use $V_0, V_1, V_2 \cdots V_{n-1}$

Ω_{na} is a sequence of functions to find Ω given $V_0 \cdots V_{n-1}$

$\Omega_{1a}(a)=a+1$
$\Omega_{2a}(a,b)=a+b$
$\Omega_{3a}(a,b,c)=ac+bc$
$\Omega_{4a}(a,b,c,d)=a\cdot c^d+bc\cdot\frac{c^d-1}{c-1}$

$\Omega_{na}(v_1,\cdots,v_n=1)=\Omega_{(n-1)a}(v_1,\cdots,v_{n-1})$

to program $\Omega_{na}(v_1,\cdots,v_n)$
Omega(vars: list of int) → int
starting from vars[0]

eg vars = [1, 2, 3]
starting at 1, step up 2 times, repeat 3 times

the previous $\Omega_{3a}(a,b,c)$ might be wrong...

$n=1$ $a+b=\Omega$

$\Omega \Rightarrow a$ $\Omega \Rightarrow b$

$n=2$ $(a+b)+b \cdots$ $a+(a+b)\cdots$

$\therefore$ actually, is $a+bc=\Omega$ $ac+b=\Omega$

$n=3$ $(a+bc)+bc\cdots = a+bcd=\Omega$

$\times$

what if substitute both into a and b

$c=1$

$(a+b)+(a+b)=2(a+b)$

$c=2$ $2((a+b)+(a+b))=4\cdot(a+b)$

$\therefore 2^c\cdot(a+b)$

$n=0$ a

$n=1$ $a+1$

$\downarrow \Omega \Rightarrow a$ for b times

$n=2$ $a+b$

$\Omega \Rightarrow a$ $\swarrow$ $\searrow$ $\Omega \Rightarrow b$ for c times

$n=3$ $a+bc$ $ac+b$

$\Omega \Rightarrow a$ $\swarrow$ $\searrow$ $\Omega \Rightarrow b$ for d times

$n=4$ $a+bcd$ $acd+b$

$\Omega \Rightarrow a$ $\swarrow$ $\searrow$ $\Omega \Rightarrow b$ for e times

$\cdots$ $acde+b$

$\Omega \Rightarrow a$ $\swarrow$ $\searrow$ $\Omega \Rightarrow b$

$\cdots$

$\searrow$ $\Omega \Rightarrow b$

$n=n$ $a+b\cdot c\cdots var_n$ $a\cdot c\cdots var_n+b$

name the functions by substitution of Ω into what,

e.g. $\Omega_{4aba}(a,b,c,d)$

so Omega(vars, subs)

$\Omega_{4aab}(a,b,c,d)=?$

$\Omega_{3aa}(a,b,c)=a+bc$

$d=2 \quad a+(a+bc)c = a+ac+bc^2$

$d=3 \quad a+ac+(a+bc)c^2 = a+ac+ac^2+bc^3$

$\therefore \Omega_{4aab}(a,b,c,d) = b\cdot c^d + \sum_{n=1}^{d} a\cdot c^{d-1} = b\cdot c^d + \frac{a\cdot(c^d-1)}{c-1}$

$H_{4aab}\{a,b,c,d,\Omega\}$ if $b\cdot c^d + \frac{a\cdot(c^d-1)}{c-1} = \Omega$

test: $\Omega_{4aab}(2,4,3,5) = 4\cdot 3^5 + \frac{2\cdot(3^5-1)}{3-1} = 1214$

omega([2,4,3,5], [0,0,1]) = 1214 ✓

$a+bc = b\cdot c$ if $a=0$

$b\cdot c^d + \frac{a\cdot(c^d-1)}{c-1} = c^d$ if $a=0$ and $b=1$

$\Omega_{3aa}(x,y,z) = \Omega_{3ab}(y,x,z)$

hopefully, all integer arithmetic can be expressed with Ω and other functions that find one variable based on others

number of Ω_n functions with given n is:

$N_1 = 1$

$N_{n+1} = N_n \cdot n$

so $N_n = (n-1)!$

omission of brackets after Ω means just $(a,b,c\cdots)$ or $(v_0, v_1, v_2 \cdots)$

$(a+b)$

$$\Omega_{2a}(x,y)=\Omega_{2a}(y,x)$$

$(a+bc)$

$$\Omega_{3aa}(x,y,z)=\Omega_{3aa}(x,z,y)$$

$(b+ac)$

$$\Omega_{3aa}(x,y,z)=\Omega_{3ab}(y,x,z)=\Omega_{3ab}(y,z,x)$$

$$\Omega_{4aaa}(x,y,z,w)=\Omega_{4aaa}(x,\overbrace{y,w,z}^{\text{any combination}})$$

$\Omega_{4aab} = b \cdot c^d + \frac{a(c^d-1)}{c-1}$

$\Omega_{3aa} = a + bc$

$\Omega_{3ab} = b + ac$

$\Omega_{4aba} = a \cdot c^d + \frac{b \cdot (c^d-1)}{c-1}$?

$d=2 \quad b+(b+ac)c = b+bc+ac^2$

$d=3 \quad b+(b+bc+ac^2)c = b+bc+bc^2+ac^3$

$d=d \quad a \cdot c^d + \sum_{n=1}^{d} b \cdot c^{d-1}$ ✓

$\Omega_{4aac}(a,b,c,d) = \Omega_{4aab}(a,c,b,d)$?

$d=2 \quad a+b(a+bc) = a+ab+b^2c$

$d=3 \quad a+b \cdot (a+ab+b^2c) = a+ab+ab^2+b^3c$

$\therefore \Omega_{4aac} = b^d \cdot c + \frac{a \cdot (b^d-1)}{b-1} \qquad \Omega_{4aab}(a,c,b,d) = b^d \cdot c + \frac{a(b^d-1)}{b-1}$ ✓

Ω_{4abc}:

$d=1 \quad b+ac$

$d=2 \quad b+a(b+ac) = b+ab+a^2c$

$d=3 \quad b+a(b+ab+a^2c) = b+ab+a^2b+a^3c$

$\therefore \Omega_{4abc} = c \cdot a^d + \frac{b \cdot (a^d-1)}{a-1}$