Hyperoperation Tree (?)

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Abstract

This is made with LaTeX, but I don't suppose it to be an actual article so I'm going to keep things informal.

1 Foreword

Traditionally, hyperoperation sequence is a sequence of binary arithmetic operators that includes addition, multiplication, exponential, etc.

But I am redefining my own from the ground up so please discard any association to the actual hyperoperation sequence to prevent confusion.

I will start with the simple step up function:

$$u(a) = a + 1$$

and I will be calling all the functions recursed from it Ω (Omega), standing for the English word Output (if you don't like it then just change it).

The number subscript means the number of time "recursed" as well as the number of parameter input and the letter subscripts mean which variable are substituted into during recursion (hold on later for more details).

The standard set of input variables for Ω is just a, b, c, and so on until the number of input matches the number subscript. And therefore I usually omit the parameters when defining the function.

$\Omega_1(a)$ or $\Omega_1 = u(a)$

2 Recursion

Repeating $u(a)$ (or Ω_1) for b times it is clear that the result is $a + b$, but the actual mechanics of recursing might not be very robustly defined (I had this mistake when I started).

And so I will define my recursion here:

Recursion in this case means, firstly, get the output of the function, then change one parameter of the function to this output, and perform the function again obtaining the output.

This is repeated for a set times.

So in this case I am recursing the output of $u(a)$ into the parameter a for b times, which I can write out in Ω form as follows:

$$\Omega_{2a}(a, b) = a + b$$

or

$$\Omega_{2a} = a + b$$

and this represents addition in traditional arithmetic.

For the next recursion, I will be recursing for c times, but now I have a choice, whether to recurse into a or b.

I won't be writing the process of getting the expression in normal arithmetic and you can find it in the other pdf where I got all my notes at.

$$\Omega_{3aa} = a + b \cdot c$$

$$\Omega_{3ab} = b + a \cdot c$$

Notice they are basically the same function but with parameter location swapped.

And Ω_{3aa} is identical to normal multiplication if we substitute 0 for a.

For recursed to the degree of 4, this is what I deduced (I am not 100% sure if it is true, but it seems like it).

$$\Omega_{4aaa} = a + b \cdot c \cdot d$$

$$\Omega_{4abb} = b + a \cdot c \cdot d$$

$$\Omega_{4aab} = b \cdot c^d + \frac{a \cdot (c^d - 1)}{c - 1}$$

$$\Omega_{4aba} = a \cdot c^d + \frac{b \cdot (c^d - 1)}{c - 1}$$

$$\Omega_{4aac} = c \cdot b^d + \frac{a \cdot (b^d - 1)}{b - 1}$$

$$\Omega_{4abc} = c \cdot a^d + \frac{b \cdot (a^d - 1)}{a - 1}$$

Still there is that sense of symmetry and recombination. I am still not sure of the mechanics yet so if anyone has the time and effort please dig into it and tell me about your results.

Note that, for Ω_{4aab}, if substituting $a = 0$ and $b = 1$, the remaining is the same as exponential.

And so I have a conjecture, that among the Ω_5 functions there is one that when $a = 0$, $b = 1$, $c = 2$, the remaining is the same as tetration (from the normal hyperoperation sequence).

However I have not dig into Ω_5 yet as of writing this so if you can do it.

Also, the relationship between number of Ω_n functions to n is:

$$N(\Omega_n) = (n - 1)!$$

3 Future Research

You might also want to look into, what if substituting into multiple parameters at once instead of only one?

Is there a way to represent any sequence of arithmetic operations as a single Ω_n function?

I'm only dealing with integers so far, what if fractions and irrationals are introduced?